A universal method for calculating and extracting the LF and RF noise behavior of nonlinear devices

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Abstract—**In this paper an efficient method for calculating the noise behavior of active devices is described. This method offers a general solution for a large variety of transistor and other devices. The method can easily be implemented in any extraction and simulation software and allows determination of LF as well as RF noise behavior. The functionality is demonstrated on a nonlinear FET model [1], [2]. Furthermore, verifications with HP series IV/ADS [3] are given for different devices.**

I. Introduction

The design of many circuits requires the extraction and the simulation of noise parameters for quite different noisy elements like FETs, BJTs or HBTs. In the past, several papers have been presented describing an extraction process for different noise models [4], [5]. Many of these methods are limited to special kinds of active devices. Most CAD circuit simulation software does not allow the extraction of parameters, although it has implemented functions to calculate noise. For noise extraction exactly these functions must be known in order to fit e.g. noise function coefficients to measurements during an optimization process. Therefore, in this paper we present an efficient method for calculating noise behavior of any device. Our proposed method is based upon the Y-matrix of any equivalent circuit and a matrix containing the noise sources for that equivalent circuit. With that knowledge both LF as well as RF noise behavior can be extracted and calculated. We describe how our proposed method can be used for extracting and simulating noisy circuits. The method can be applied to any extraction and simulation software.

II. Transforming noise sources

When simulating a circuit, noisy n-ports must be transformed into a non-noisy n-port and associated noise sources at outer ports. Fig. 1 shows an example of transforming noisy networks to non-noisy ones including a current source and a voltage source at the output for the 2-port. Using the transformation matrix

$$[T]^{(y\to a)} = \begin{bmatrix} 0 & -\frac{1}{\underline{y}_{21}} \\ 1 & -\frac{\underline{y}_{11}}{\underline{y}_{21}} \end{bmatrix}, \qquad (1)$$

the noise matrix of currents can be calculated in a form

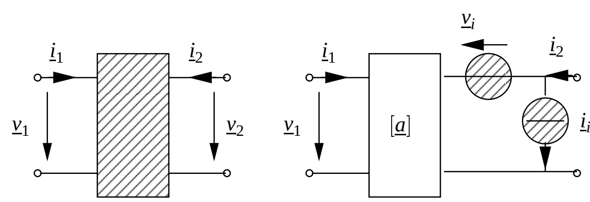

Fig. 1. Transforming noise sources to outer ports.

using equation (2):

$$\begin{bmatrix} v_{\mathrm{i}}^{(a)} \\ i_{\mathrm{o}}^{(a)} \end{bmatrix} = [T]^{(y\to a)} \begin{bmatrix} i_{\mathrm{i}}^{(y)} \\ i_{\mathrm{o}}^{(y)} \end{bmatrix}, \qquad (2)$$

whereby the indices i and o describe input and output ports. For calculating noise power, the column vectors are multiplied with the conjugated row vectors of the noise source matrix and end up by division with $4kT\Delta f$ in the wellknown correlation matrix

$$\begin{aligned} [\underline{C}]^{(a)} &= \frac{1}{4kT\Delta f}\left(\begin{bmatrix} \underline{v}_{\mathrm{i}} \\ \underline{i}_{\mathrm{i}} \end{bmatrix} \begin{bmatrix} \underline{v}_{\mathrm{i}}^* & \underline{i}_{\mathrm{i}}^* \end{bmatrix}\right) \\ &= \frac{1}{4kT\Delta f} \begin{bmatrix} \underline{v}_{\mathrm{i}}\underline{v}_{\mathrm{i}}^* & \underline{v}_{\mathrm{i}}\underline{i}_{\mathrm{i}}^* \\ \underline{i}_{\mathrm{i}}\underline{v}_{\mathrm{i}}^* & \underline{i}_{\mathrm{i}}\underline{i}_{\mathrm{i}}^* \end{bmatrix}. \end{aligned} \qquad (3)$$

III. Noise analysis

There are different ways of calculating noise. One way is the division of a circuit into different sub-2-ports with noise sources, that have been transformed as described above. Noise behavior for each 2-port can be easily calculated e.g. using node analysis. The separately calculated 2-ports can be combined using the transformation matrices.

In this paper a different approach is used. Taking into account the Tellegen theorem, current and voltage sources can be transformed to the outer ports of a network N. To do this, the adjuncted network $\widehat{N}$ is required, which has a definite reversible relation to the network N. The Y-matrix of the adjuncted network $\widehat{N}$ represents the transposed Y-matrix of the network N as well. With that knowledge, a current source e.g. can be transformed to the outer ports using just one node analysis for the adjuncted network and $k-1$ analysis for the network N,

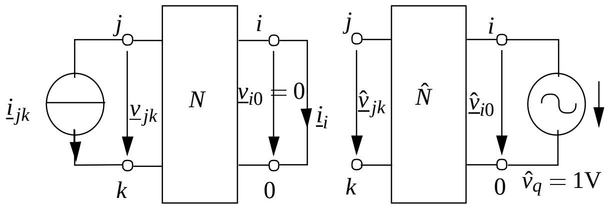

Fig. 2. Circuitry of networks for using the Tellegen theorem.

whereby k gives the number of nodes in the network N. Using a circuitry as shown in fig. 2, the current transformation function α_i is equal to the negative voltage transformation function of the adjuncted network $\widehat{N}$:

$$\alpha_{\mathrm{i},jk} = \frac{i_\mathrm{i}}{i_{jk}} = -\frac{\hat{v}_{jk}}{\hat{v}_\mathrm{q}}. \tag{4}$$

This fact can be used for calculating noise behavior as shown for an example in the next section.

IV. Calculating FET noise

The applicability of the Tellegen theorem is shown for a FET equivalent circuit (fig. 3). The circuitry takes care of different noise mechanisms in FETs, like resistive noise and channel noise. Furthermore, $1/f$ noise is taken into account. The ten nodes of the equivalent circuit are numbered in order to determine the 10×10 Y-matrix. In correspondence with the Y-matrix, a 10×10 noise matrix can be defined. The position of noise sources in that matrix is given by the node numbers at the end of the noise source. The direction of the current is defined in the way, that at matrix entry $i_{\mathrm{j,k}}$ the noise current in the equivalent circuits flows from node j into the direction of node k. All noise current sources have only positive entries. The factor $\sqrt{4kT\Delta f}$ is placed outside the brackets. For the presented equivalent circuit the noise current matrix is given to

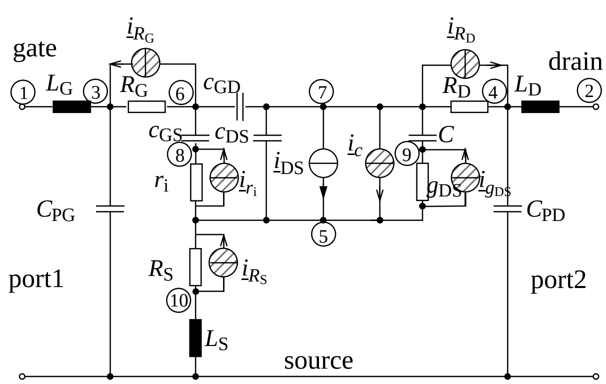

Fig. 3. Noisy equivalent circuit for a FET.

$$[i_\mathrm{N}] = \sqrt{4kT\Delta f}\cdot$$
$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & i_{5,8} & i_{5,9} & 0 \\
0 & 0 & i_{6,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & i_{7,4} & i_{7,5} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & i_{10,5} & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{5}$$

with the resistive noise

$$\begin{aligned}
i_{5,8} &= \sqrt{\frac{1}{r_\mathrm{i}}}, \\
i_{5,9} &= \sqrt{g_\mathrm{DS}}, \\
i_{6,3} &= \sqrt{\frac{1}{R_\mathrm{G}}}, \\
i_{7,4} &= \sqrt{\frac{1}{R_\mathrm{D}}},
\end{aligned}$$

$$\begin{aligned}
i_{7,5} &= \sqrt{g_{\mathrm{i_c}}}, \\
i_{10,5} &= \sqrt{\frac{1}{R_\mathrm{S}}}
\end{aligned}$$

including the channel noise

$$g_{\mathrm{i_c}} = \frac{T_\mathrm{sim}}{T_0}\frac{2}{3}g_\mathrm{m} + k_f\frac{I_c^{af}}{f^b 4kT_0}. \tag{6}$$

The next calculation steps are converting the Y-matrix into an Z-matrix, and re-calculating the 2×2 Y-matrix because of the examined 2-port and computing the transposed Y-matrix

$$[\underline{y}_T] = [\underline{y}]^T. \tag{7}$$

The adjuncted network shown in fig. 4 is used in order to calculate a voltage transformation function. The volt-

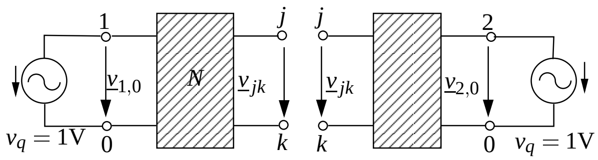

Fig. 4. Using the Tellegon theorem for noisy circuits.

age source v_q can be transformed into a current source as shown in equation (8)

$$
\begin{aligned}
\underline{i}_{Q_1} &= \frac{v_q}{\underline{y}_{T,2\times2_{1,1}}}, \\
\underline{i}_{Q_2} &= \frac{v_q}{\underline{y}_{T,2\times2_{2,2}}}.
\end{aligned} \tag{8}
$$

Using the Gauß algorithm, the equation system containing the transposed Y-matrix $[\underline{y}_T]$ can be solved for the voltages:

$$
\begin{bmatrix} i_{Q_1} \\ \vdots \\ i_{Q_{10}} \end{bmatrix} = \begin{bmatrix} y_{T_{1,1}} & \cdots & y_{T_{1,10}} \\ \vdots & \ddots & \vdots \\ y_{T_{10,1}} & \cdots & y_{T_{10,10}} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_{10} \end{bmatrix}. \tag{9}
$$

The result delivers a voltage transformation factor, that is equal to the negative current transformation factor. Now, for each noise source the influence at the input and output port of the example 2-port can be calculated. Taking care of the correlation of the noise sources, the influences of the single noise sources can be added to one single noise source at the input as well as the output. At this point a noise free 2-port with two outer noise current sources in Y representation has been determined. With the two noise sources, the correlation matrix can be calculated as described in equation (11). With the help of the transformation matrix $[T]^{(y\to a)}$, the correlation matrix can be changed into the following representation:

$$
[T]^{(y\to a)} = \begin{bmatrix} 0 & -\frac{1}{\underline{y}_{2\times2_{21}}} \\ 1 & -\frac{\underline{y}_{2\times2_{11}}}{\underline{y}_{2\times2_{21}}} \end{bmatrix}, \tag{10}
$$

$$
[\underline{C}]^{(a)} = [T]^{(y\to a)}[\underline{C}]^{(y)}[T]^{(y\to a)+}. \tag{11}
$$

Now, the noise figure can be calculated to

$$
F = 1 + \frac{T_\mathrm{sim}}{T_0}\frac{|\underline{y}_\mathrm{G}|^2 C_{11}^{(a)} + C_{22}^{(a)} + 2\Re\{\underline{y}_\mathrm{G}\underline{C}_{12}^{(a)}\}}{g_\mathrm{G}}, \tag{12}
$$

whereby T_{sim} is the simulation temperature and $\underline{y}_\mathrm{g}$ the generator admittance. The noise resistor is equal to

$$
R_\mathrm{n} = \frac{T_\mathrm{sim}}{T_0}C_{11}^{(a)} \tag{13}
$$

and the optimum reflection coefficient is

$$
\Gamma_{\mathrm{G_{opt}}} = \frac{1-\underline{y}_{\mathrm{G_{opt}}}Z_0}{1+\underline{y}_{\mathrm{G_{opt}}}Z_0}. \tag{14}
$$

The $1/f$ noise can be calculated using the following equation:

$$
V_\mathrm{NF}[\mathrm{dBm}/\sqrt{\mathrm{Hz}}] =
$$

$$
20\log\left(\sqrt{\frac{T_\mathrm{sim}}{T_0}\cdot i_\mathrm{o} i_\mathrm{o}^*}\,\frac{50\,\Omega}{50\,\Omega\,\Re(\underline{y}_{2\times2_{22}})+1}\cdot 1000\right). \tag{15}
$$

V. Verifications

In this section some verifications of the proposed method for calculating and extracting noise are presented. Fig. 5

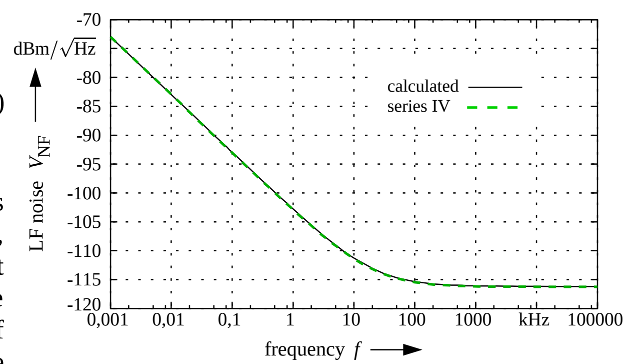

Fig. 5. Comparison of calculated $1/f$ noise for a 4-finger 50 μm HEMT at a bias point of $V_\mathrm{GS} = -0.2$ V and $V_\mathrm{DS} = 2$ V.

shows the $1/f$ noise for a 4-finger 50 μm HEMT at a bias point of $V_\mathrm{GS} = -0.2$ V and $V_\mathrm{DS} = 2$ V. The calcula-

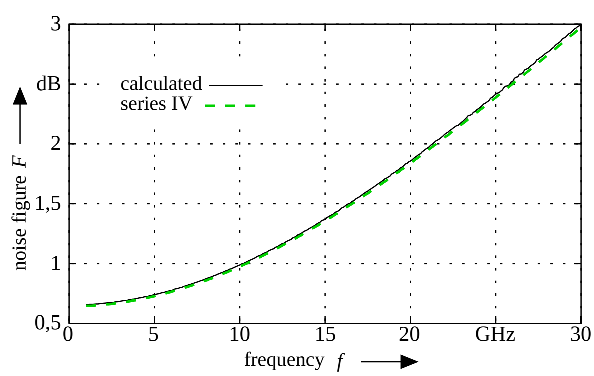

Fig. 6. Comparison of noise figure for a 4-finger 50 μm HEMT at a bias point of $V_\mathrm{GS} = 0$ V and $V_\mathrm{DS} = 2$ V.

tion with our proposed method is compared to the one of the circuit simulation HP series IV internal model. Both calculation results are identical. In fig. 6 calculations for the same device can be seen for the noise figure at a bias point of $V_{GS} = 0$ V and $V_{DS} = 2$ V. Again, both results are in excellent agreement. The comparison with measure-

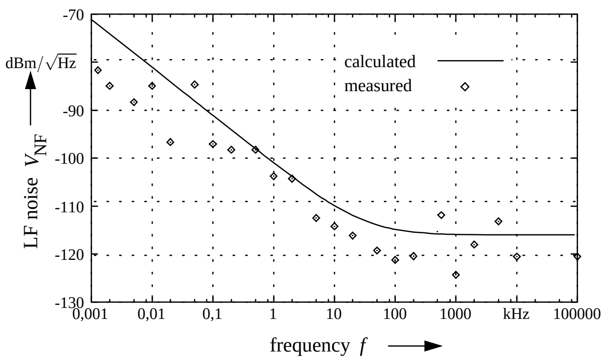

Fig. 7. Comparison of simulated and measured $1/f$ noise for a 4-finger 50 μm HEMT device at a bias point of $V_{GS} = 0$ V and $V_{DS} = 1.5$ V.

ments can be seen in the next two plots (fig. 7 and fig. 8). The same proposed algorithm was used to extract parameters for the $1/f$ noise as described in equation (16)

$$i_f^2 = k_f \frac{I^{af}}{f^b} \Delta f \tag{16}$$

for a 4-finger 50 μm HEMT device at a bias point of $V_{GS} = 0$ V and $V_{DS} = 1.5$ V. The extracted parameters are

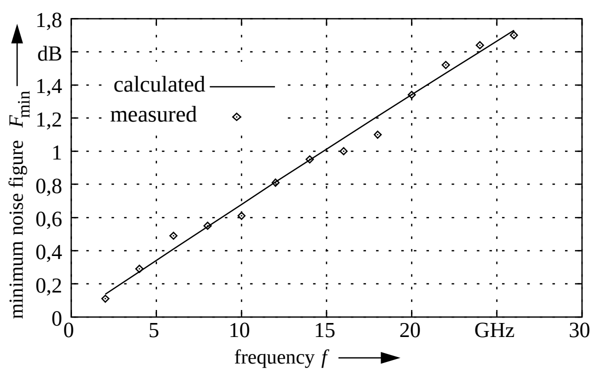

Fig. 8. Comparison of measured and simulated minimum noise figure for a 4-finger 40 μm HEMT at a bias point of $V_{GS} = -0.2$ V and $V_{GS} = 2$ V.

$k_f = 1.3$ e^{-12}, $af = 0.9$ and $b = 1.05$. Fig. 8 shows the comparison of the simulated and measured noise figure for a 4-finger 40 μm HEMT at a bias point of $V_{GS} = -0.2$ V and $V_{GS} = 2$ V. All simulations are in excellent agreement with measurements.

VI. Conclusions

In our paper an effective method for calculating both LF as well as RF noise of active devices has been demonstrated. The method was compared with the one used by HP-EEsof libra series IV circuit simulation software. It has been shown that both methods lead to nearly the same results. Furthermore, the method we used can easily be applied to any extraction software in order to extract e.g. coefficients for the $1/f$ noise behavior. Also, simulation results in excellent agreement to measurements have been shown.

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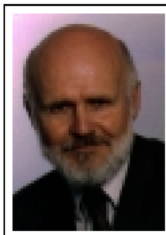

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